

# Logik für Informatiker

## Logic for computer scientists

### Proof rules for quantifiers

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# Universal Elimination

## ( $\forall$ Elim)

$$\begin{array}{c|l} \triangleright & \forall x S(x) \\ & \vdots \\ & S(c) \end{array}$$

# Existential Introduction

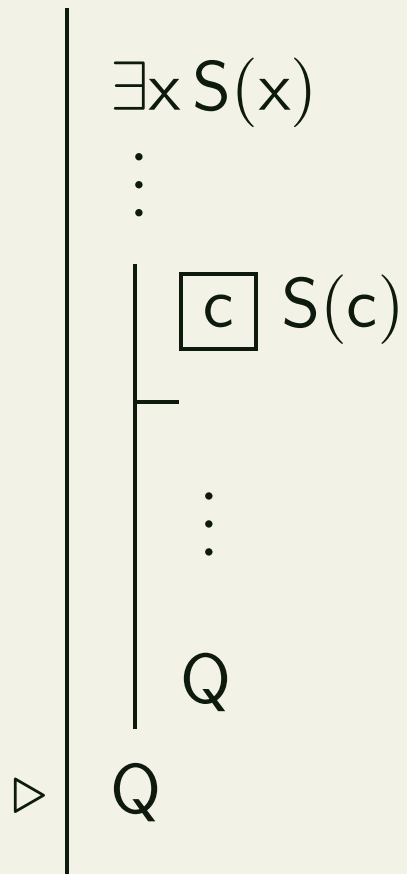
## $(\exists \text{ Intro})$

$$\begin{array}{l} \triangleright \left| \begin{array}{l} S(c) \\ \vdots \\ \exists x S(x) \end{array} \right. \end{array}$$

## Example: $\forall$ -Elim and $\exists$ -Intro

$\forall x[\text{Cube}(x) \rightarrow \text{Large}(x)]$   
 $\forall x[\text{Large}(x) \rightarrow \text{LeftOf}(x, b)]$   
 $\text{Cube}(d)$   
—  
 $\exists x[\text{Large}(x) \wedge \text{LeftOf}(x, b)]$

## Existential Elimination ( $\exists$ Elim):

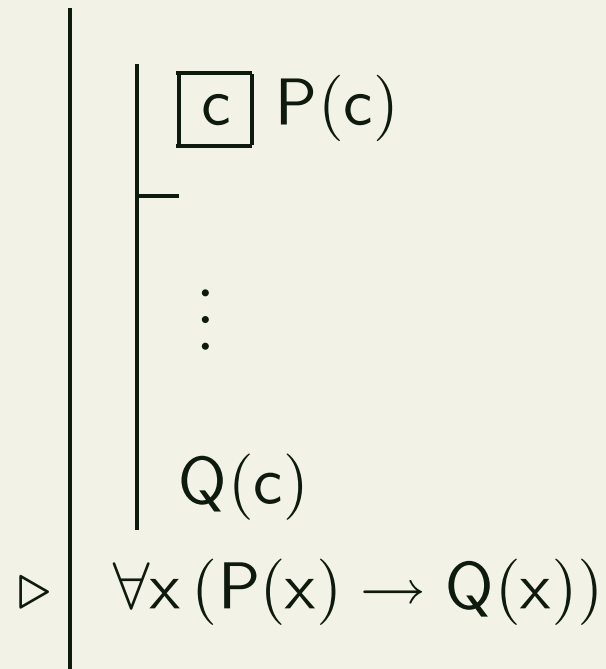


Where  $c$  does not occur outside the subproof where it is introduced.

## Example: $\exists$ -Elim

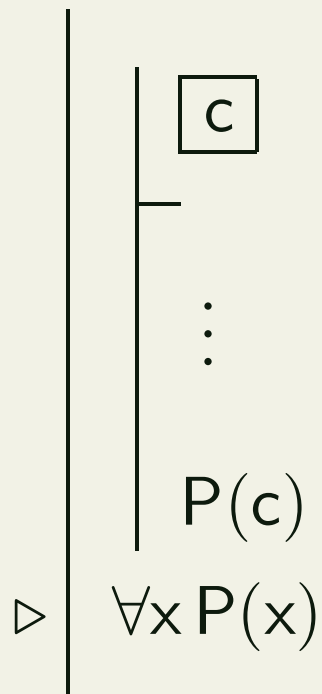
$$\begin{array}{|l} \forall x[\text{Cube}(x) \rightarrow \text{Large}(x)] \\ \forall x[\text{Large}(x) \rightarrow \text{LeftOf}(x, b)] \\ \exists x \text{ Cube}(x) \\ \hline \exists x[\text{Large}(x) \wedge \text{LeftOf}(x, b)] \end{array}$$

## General Conditional Proof ( $\forall$ Intro):



Where  $c$  does not occur outside the subproof where it is introduced.

## Universal Introduction ( $\forall$ Intro):



Where  $c$  does not occur outside the subproof where it is introduced.

## Example: $\forall$ -Intro

$$\left| \begin{array}{l} \exists y[\text{Girl}(y) \wedge \forall x(\text{Boy}(x) \rightarrow \text{Likes}(x, y))] \\ \hline \forall x[\text{Boy}(x) \rightarrow \exists y(\text{Girl}(y) \wedge \text{Likes}(x, y))] \end{array} \right.$$

## Example: de Morgan's Law

$$\left| \begin{array}{l} \neg \forall x P(x) \\ \hline \exists x \neg P(x) \end{array} \right|$$

(is not valid in intuitionistic logic, only in classical logic)

## Example: The Barber Paradox

$$\begin{array}{|l} \exists z \exists x [ManOf(x, z) \wedge \forall y (ManOf(y, z) \rightarrow \\ \quad (Shave(x, y) \leftrightarrow \neg Shave(y, y)))] \\ \hline \perp \end{array}$$

# Identity Introduction (= Intro)

$$\triangleright \mid n = n$$

## Identity Elimination (= Elim)

|                  |          |
|------------------|----------|
|                  | $P(n)$   |
|                  | $\vdots$ |
|                  | $n = m$  |
|                  | $\vdots$ |
| $\triangleright$ | $P(m)$   |

# Reflexivity, symmetry and transitivity

$$\vdash \forall x \, x = x$$

$$\vdash \forall x \, \forall y \, x = y \rightarrow y = x$$

$$\vdash \forall x \, \forall y \, \forall z \, (x = y \wedge y = z) \rightarrow x = z$$