1. Periods Hierarchy

1.1. **Theories.**

- `periods_root`
  - definably equivalent to: `ap_root`
- `periods`
  - nononconservative extension of: `periods_root`
  - relatively interprets: `ap_root`
- `mixed_periods`
  - nononconservative extension of: `periods_root`
  - definably equivalent to: `ap_interval`
  - relatively interprets: `ap`
- `periods_over_integers`
  - nononconservative extension of: `mixed_periods`
- `periods_over_rationals`
  - nononconservative extension of: mixed_periods
  - definably equivalent to: `ap_rational`

1.2. **Translation Definitions.**

Definition 1. *The translation definitions Σ_{p_ap} for the interpretation of theories in $\mathbb{H}_{Approximate\_Point}$ to theories in $\mathbb{H}_{Periods}$:*

```
(forall (x y) (iff  (precedence x y) (precedes x y)))
(forall (x y) (iff  (inclusion x y) (finer x y)))
(forall (x y) (iff  (overlaps x y) (ncdf x y)))
```

2. *Approximate_Point* Hierarchy

2.1. **Theories.**

- `ap_root`
  - definably equivalent to: `periods_root`
- `ap`
  - nonconservative extension of: `ap_root`
  - relatively interprets: `meets_root`
- `m_exist`
  - nonconservative extension of: `ap`
  - relatively interprets: `im`
- `ap_interval`
  - nonconservative extension of: `ap`
  - definably equivalent to: `mixed_periods`
- `ap_discrete`
  - nonconservative extension of: `ap`
- `ap_integer`
  - nonconservative extension of: `ap_discrete`
  - nonconservative extension of: `ap_interval`
  - relatively interprets: `periods_over_integers`
- `ap_dense`
  - nonconservative extension of: `ap`
- `ap_rational`
  - nonconservative extension of: `ap_dense`

     – nonconservative extension of: `ap_interval`
     – relatively interprets: `periods_over_rationals`

2.2. Translation Definitions.

Definition 2. *The translation definitions Σ_{ap_p} for the interpretation of theories in $\mathbb{H}_{Periods}$ to theories in $\mathbb{H}_{Approximate-Point}$:*

```
(forall (x y) (iff  (precedence x y) (precedes x y)))
(forall (x y) (iff  (inclusion x y) (finer x y)))
(forall (x y) (iff  (overlaps x y) (ncdf x y)))
```

The translation definitions Σ_{ap_p} for the interpretation of theories in $\mathbb{H}_{Periods}$ to theories in $\mathbb{H}_{Approximate-Point}$ is equivalent to Σ_{p_ap}.

Definition 3. *The translation definitions Σ_{ap_im} for the interpretation of theories in $\mathbb{H}_{Interval-Meeting}$ by theories in $\mathbb{H}_{Approximate-Point}$ is the set of sentences*

```
(forall (x y) (iff  (meets x y)
                    (and  (precedes x y)
                          (not  (exists (z)
                                        (and  (precedes x z)
                                        (precedes z y)))))))
```

3. *Interval_Meeting* HIERARCHY

3.1. Theories.

- `im`
  - nonconservative extension of: `meets_root`
  - relatively interprets: `ap_root`
- `allen_hayes`
  - nonconservative extension of: `im`
- `ladkin_intq`
  - nonconservative extension of: `allen_hayes`
  - definably equivalent to: `ap_rational`

3.2. Translation Definitions.

Definition 4. *The translation definitions Σ_{im_ap} for the interpretation of theories in $\mathbb{H}_{Approximate\_Point}$ by theories in $\mathbb{H}_{Interval\_Meeting}$:*

```
(forall (x y) (iff  (precedes x y)
                    (or  (meets x y)
                         (exists (z)
                             (and  (meets x z)
                             (meets z y))))))

(forall (x y) (iff  (finer x y)
                    (or  (starts x y) (during x y) (finishes x y) (= x y))))
```